Minitab

SPC

LESSON: Cardboard Thickness - Homework

Homework 4 NAME:____

Topics: Xbar-R charts with Minitab

Solve the following problems and show your work.

Constructing $\overline{X}$ and R charts given sample data; i.e., μ and σ are not given – they will be estimated from the given data for the process.

A paper manufacturing company must control the thickness of cardboard sheets it produces. Random samples of size **n = 5 sheets** are selected each hour and the thickness is recorded for **k = 30 subgroups** of n = 5 sheets. Each hour, the n = 5 sample observations are placed in **Columns C1-C5 across a row**. Each row in the worksheet constitutes one hour. The **specifications** on the thickness of the cardboard sheets are **0.5 ± 0.04 mm**. [Note: Specifications ("specs") are provided by the customer – these are NOT the $\overline{X}$ control limits for the process.]

This data is in the Minitab worksheet entitled **Hmwk4DATA_CardboardThickness**.

Hour	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5
1	0.52	0.55	0.49	0.51	0.52
2	0.53	0.5	0.51	0.51	0.5
3	0.52	0.51	0.55	0.5	0.52
4	0.42	0.45	0.43	0.42	0.46
5	0.48	0.5	0.47	0.51	0.51
...	...	...	...	...	...

Determine the center line for the $\overline{X}$ chart “manually” in Minitab by following these instructions.

(a) Have Minitab determine the subgroup mean for each row of n = 5 observations.

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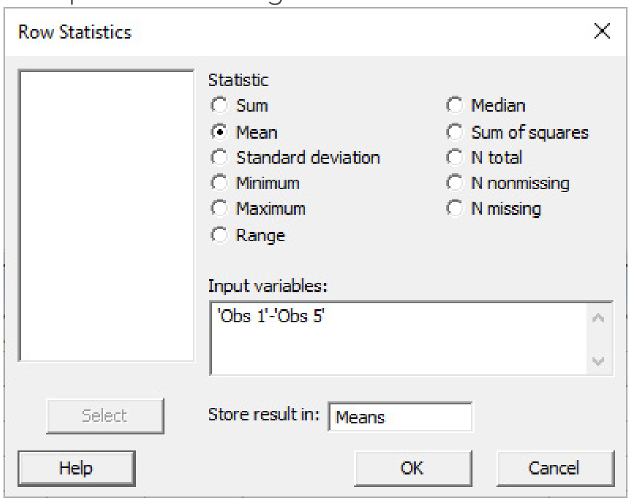
Choose **Calc** > **Row Statistics**.

Complete the dialog box as shown below



Click **OK**.

Thus, C7 contains the mean for each subgroup of size n = 5.

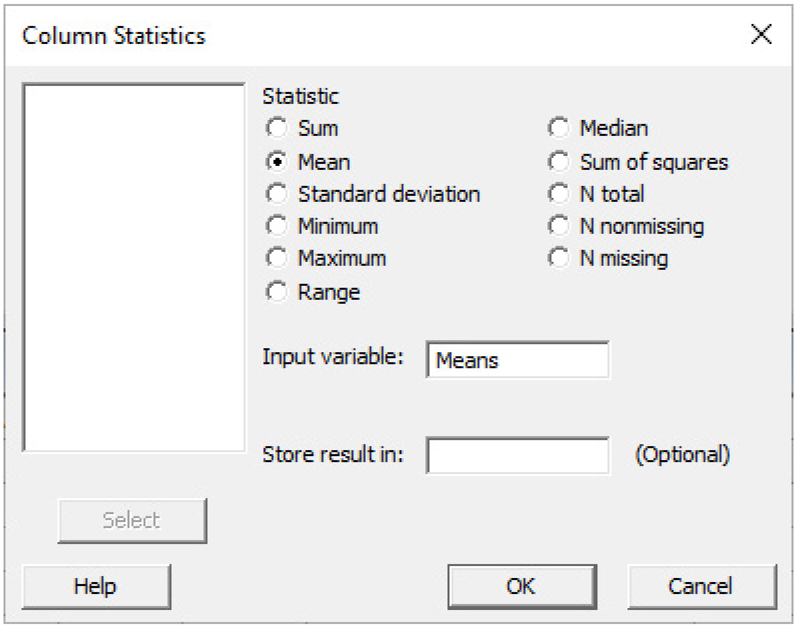
List the next 4 subgroup means (correct to 3 decimal places):

Hour	1:	0.518
	2:	0.510
	3:	0.520
	4:	0.436
	5:	0.494

(b) In Minitab, determine the “mean of the means,” or what we call $\bar{\bar{X}}$. The **center line** of the $\bar{\bar{X}}$ chart is $\bar{\bar{X}}$.

Minitab

Choose **Calc** > **Column Statistics**.
Complete the dialog box as shown below



Click **OK**.

What is the value of $\bar{\bar{X}}$?
0.501

(c) Using the sample means $\bar{X}_1$, $\bar{X}_2$, X_3,...,$\bar{X}_{30}$, write down the mathematical formula used to compute $\bar{X}$.

$\bar{X}$= **Solution**

$$\bar{\bar{X}} = \sum_{i=1}^{30} \frac{\bar{X}_i}{30} = \frac{1}{30}\sum_{i=1}^{30} \bar{X}_i = \frac{\bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_{30}}{30}$$

In order to compute the Lower Control Limit (LCL) and the Upper Control Limit (UCL) for the $\bar{X}$ chart, we next need to compute $\bar{R}$.

(d) First, determine the ranges for each row of n = 5 observations.

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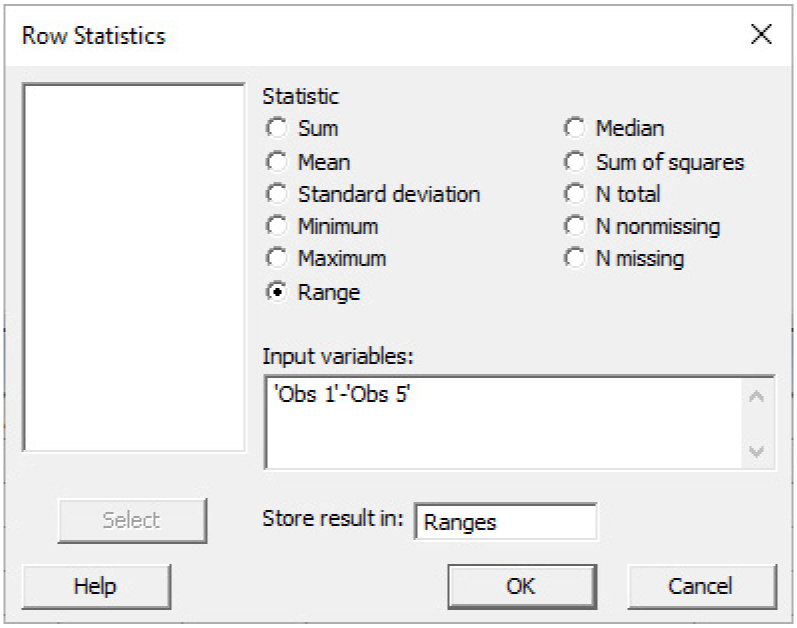
Choose **Calc** > **Row Statistics**.



Click **OK**.

Thus, C8 contains the range for each subgroup of size n = 5.

List the next 4 subgroup ranges (correct to 2 decimal places):

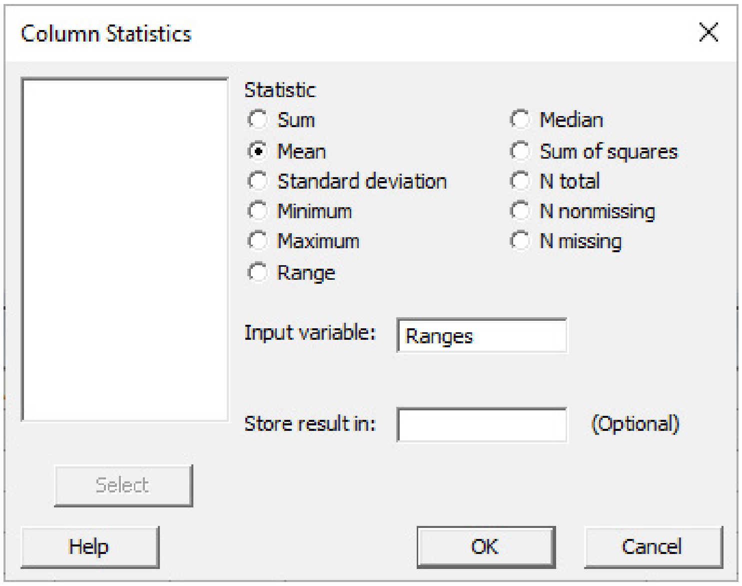
Hour	1:	0.06
	2:	**0.03**
	3:	**0.05**
	4:	**0.04**
	5:	**0.04**

(e) Next determine the "mean of the ranges," or what we call $\overline{R}$. The center line of the R chart is $\overline{R}$.

Minitab

Choose **Calc** > **Column Statistics**.

Complete the dialog box as shown below.



Click **OK**.

What is the value of $\overline{R}$?

Solution: 0.0547

(f) Using the sample ranges, R1, R2, ..., R30, write down the mathematical formula used to compute $\overline{R}$.

$\overline{R}$= Solution

$$\overline{R} = \sum_{i=1}^{30} \frac{R_i}{30} = \frac{1}{30}\sum_{i=1}^{30} R_i = \frac{R_1 + R_2 + \cdots + R_{30}}{30}$$

(g) Now that we have values for $\overline{\overline{X}}$ and R, we can compute the LCL and UCL for the $\overline{X}$ and R charts. Show your work in determining values for the LCL and UCL of the $\overline{X}$ chart. **Hint**: See the **QualityMethods_ FormulasReference_2019** resource for Chart Formulas and Control Chart constants. Recall that the subgroup size is n = 5.

Solution: UCL for $\bar{X}$ chart: $\bar{\bar{X}} + A_2\bar{R} \cong 0.5013 + \mathbf{0.577} \cdot 0.0547 \cong \mathbf{0.5329\ mm}$

or $\bar{\bar{X}} + 3 \cdot \frac{\frac{\bar{R}}{d_2}}{\sqrt{n}} \cong 0.5013 + 3 \cdot \frac{0.0547/2.326}{\sqrt{5}} \cong \mathbf{0.5329\ mm}$

LCL for $\bar{X}$ chart: $\bar{\bar{X}} - A_2\bar{R} \cong 0.5013 - \mathbf{0.577} \cdot 0.0547 \cong \mathbf{0.4697\ mm}$

or $\bar{\bar{X}} - 3 \cdot \frac{\frac{\bar{R}}{d_2}}{\sqrt{n}} \cong 0.5013 + 3 \cdot \frac{0.0547/2.326}{\sqrt{5}} \cong \mathbf{0.4697\ mm}$

(h) Compute the LCL and UCL for the R chart. Again, write down your calculations and determine the LCL and UCL.

Solution:

UCL for R chart: $\text{UCL}_{\bar{R}}$: $D_4\bar{R} \cong 2.115 \cdot 0.0547 \cong$ **0.1156 mm**

LCL for R chart: $\text{LCL}_{\bar{R}}$: $D_3\bar{R} \cong 0 \cdot 0.0547 =$ **0 mm**

(i) Use Minitab to construct the $\overline{X}$ and R control charts for hourly subgroups of size n = 5. The center lines and control limits should match the values you computed "by-hand."

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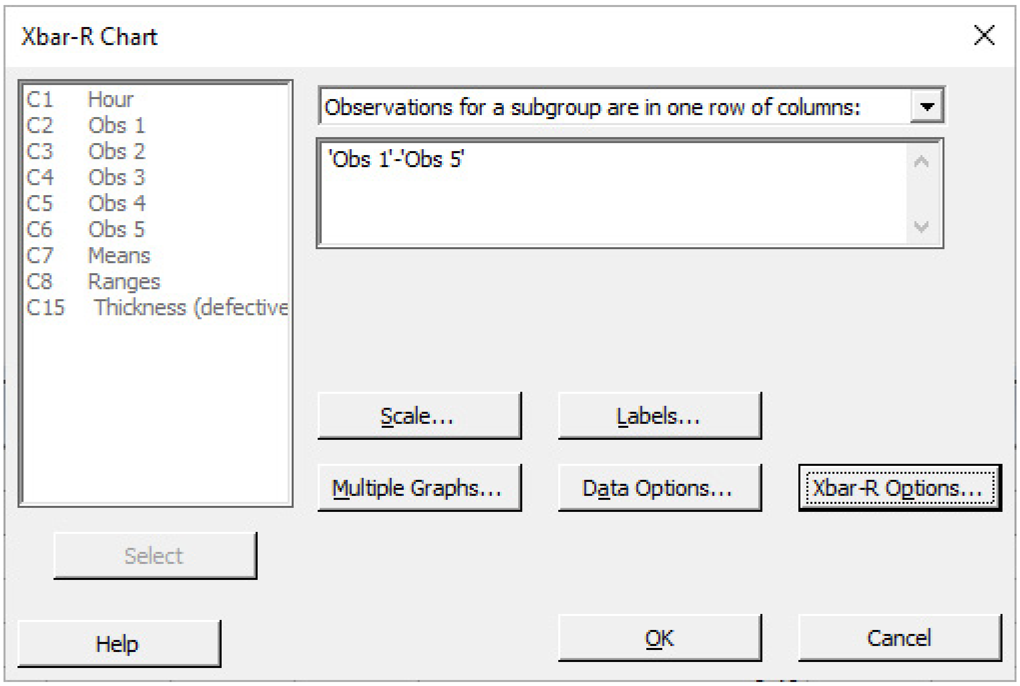
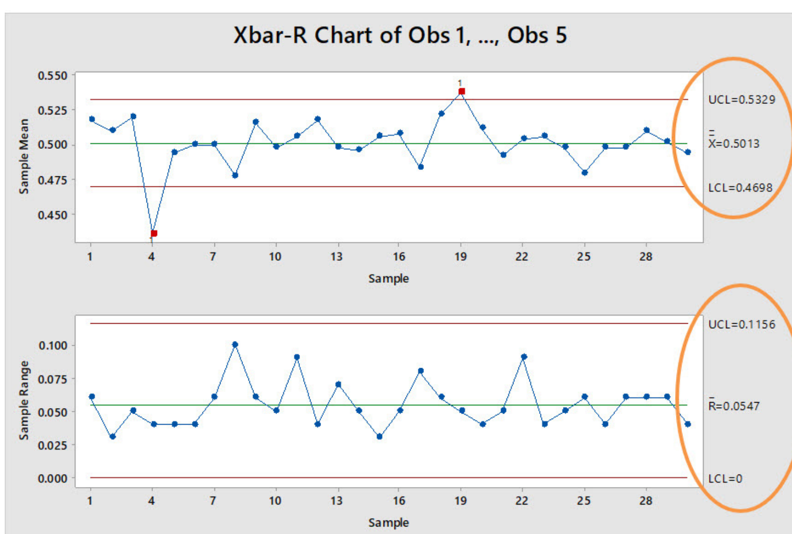
Choose **Stat** > **Control Charts** > **Variables Charts for Subgroups** > **Xbar-R**.
From the drop-down menu, select "Observations for a subgroup are in one row of columns:" Complete the dialog box as shown below



Click **Xbar-R Options**.
Click **Tests** and select **Perform all tests for special causes**.
Click **OK** in each dialog box.



(j) Is the process in control if all rules are applied? If not, **what subgroup number(s)** have violated the "out of control" rules and **which rule(s)** are violated? This information is available in the Output Window.

Solution: No – the process is not in control when all rules are applied. Samples 4 and 19 both violate TEST RULE 1: One point beyond the UCL or LCL. Grading: This part is not worth any points

Test Results for Xbar Chart of Obs 1, ..., Obs 5
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 4, 19

(k) Assuming special cause variation for the out-of-control points (e.g., power failure, defective material), "remove" them and find the revised control limits.

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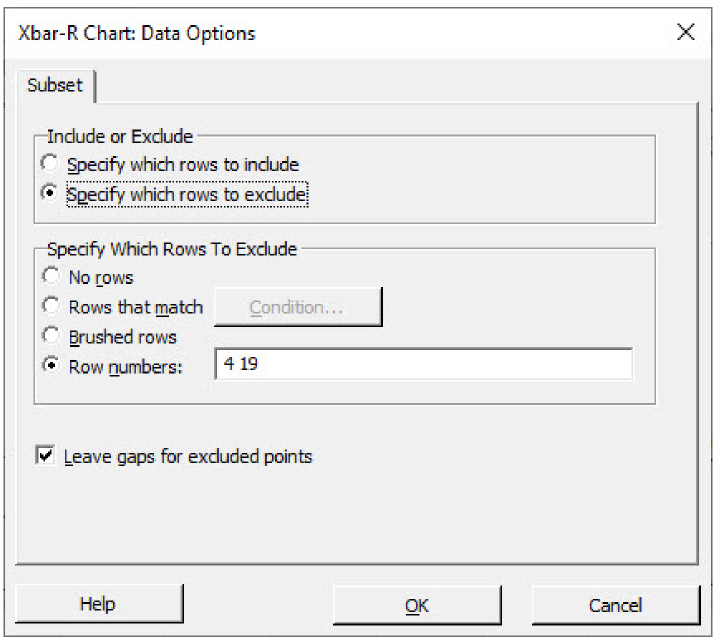
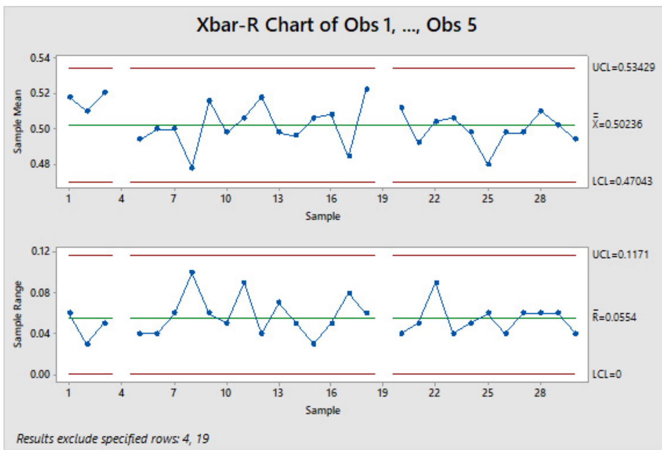
Choose **Stat > Control Charts > Variables Charts for Subgroups > Xbar-R**.
Choose **Data Options**.
Choose **Row numbers** and complete the dialog box as shown below.



Click **OK** in each dialog box.

	$\overline{X}$ Control Chart	R Control Chart
Centerline	**0.5024**	0.0554
Upper Control Limit	**0.5343 [+0.5]**	**0.1771 [+0.5]**
Lower Control Limit	**0.4704**	0

(k) (continued) Record the **new center line and control limits** for the $\overline{X}$ and R charts as shown on the new Minitab control charts.



(l) Now that the out of control points are removed and your process is in control, what is the estimate of the process mean $\hat{\mu}$ and the process standard deviation $\hat{\sigma}$?

Solution: Estimate of process mean: $\hat{u} = \mathbf{0.5024}$

Estimate of process standard deviation: $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.0554}{2.326} \cong \mathbf{0.0238}$

(m) "Hypothetically" what proportion of the output will be nonconforming? That is, what proportion of the cardboard thicknesses (according to their distribution with mean $\hat{\mu}$ and the process standard deviation $\hat{\sigma}$ from part (l)) is expected to be beyond the specification limits?

Recall: The **specifications** on the thickness are **0.5 ± 0.04 mm**

Solution: Let X represent the thickness of this company's cardboard sheets. Then from part (l), and assuming thicknesses are normally distributed, X ~ Normal (0.5024, 0.0238). We want to determine P(X < 0.46) + P(X > 0.54), or equivalently, 1 – P(0.46 < X < 0.54).

$$1 - P(0.46 < X < 0.54) = 1 - P\left(\frac{0.46-0.5024}{0.0238} < \frac{X-0.5024}{0.0238} < \frac{0.54-0.5024}{0.0238}\right) = 1 - P(-1.78 < Z < 1.58) \cong 0.0945.$$

Using Minitab, the solution also rounds to **0.0945**.

(n) What assumption(s) did you make about the thickness of cardboard sheets to do part (m) to determine this proportion?

Solution: In order to perform the computation in part (m), we had to assume the **cardboard thicknesses are normally distributed**.

(o) Check the assumption referred to in part (n) using the data with the out of control points removed. I put the single observations with the out of control points removed in column C15. What p-value accompanies this assumption? What's your decision regarding this assumption?

Solution: Unfortunately, the p-value for the normality test is **< 0.005** using the **Anderson-Darling test**. This implies that the **data does not come from a normally distributed population**. Strangely (rare), the p-value for the normality test is > 0.100 using the Ryan-Joiner test. This implies the data **does come from a normally distributed population**. If you used AD, reject the assumption of normality; if you used RJ, do not reject the assumption of normality.

As we discussed in class, the data is normally distributed, but the discrete nature of the data makes it appear not normally distributed according to the AD test. Adding a 3rd decimal place to the data does give us a p-value greater than 0.05 when applying the AD test to the data.